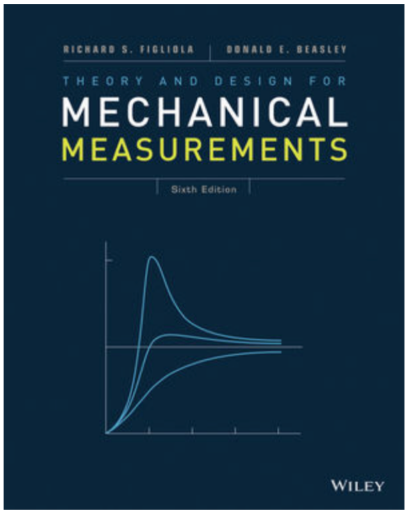


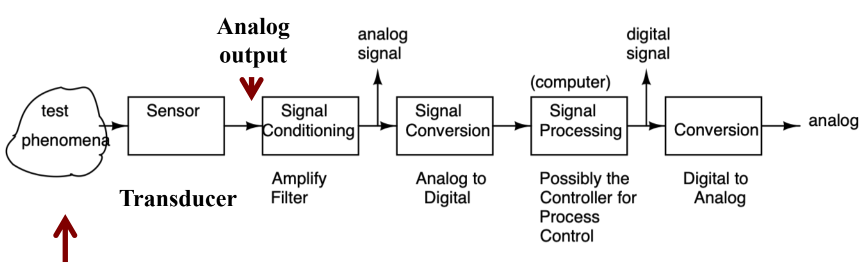
Analog and Digital Filtering

Engr325

Instrumentation

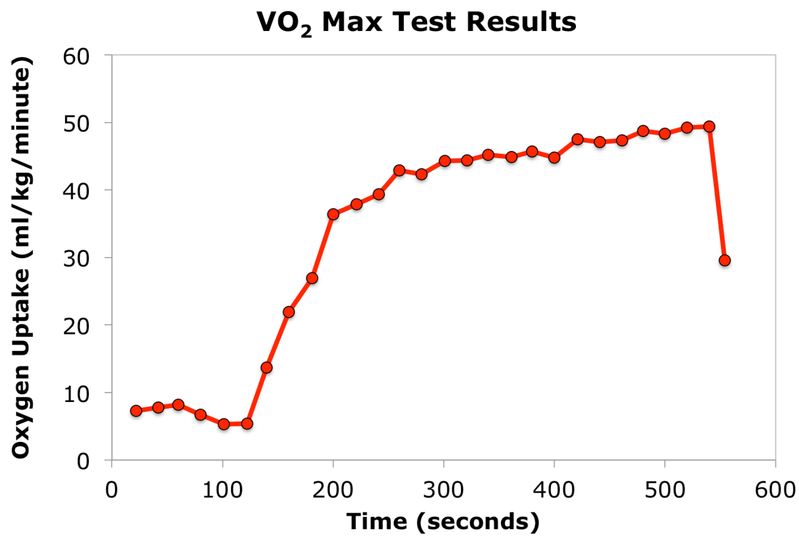
Dr Curtis Nelson

Instrumentation System

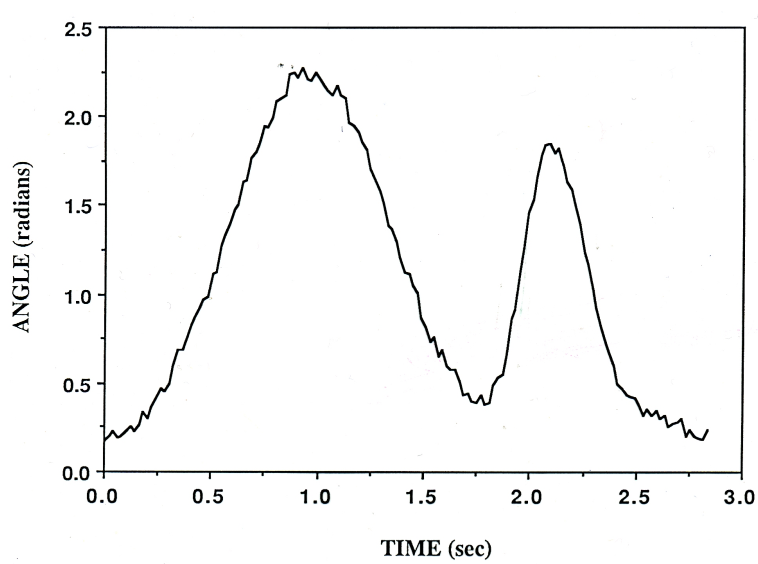


Temperature, Light, Displacement, Position, Vibration, Strain, Angle, Velocity, Acceleration, Flow rate, Pressure, Viscosity, Heart Rate, Etc.

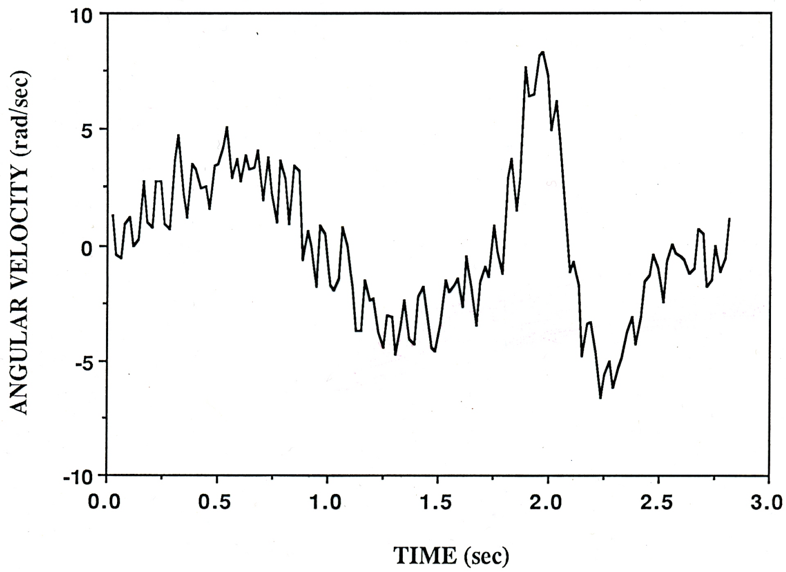
Noisy Signals are Found in Many Measurements



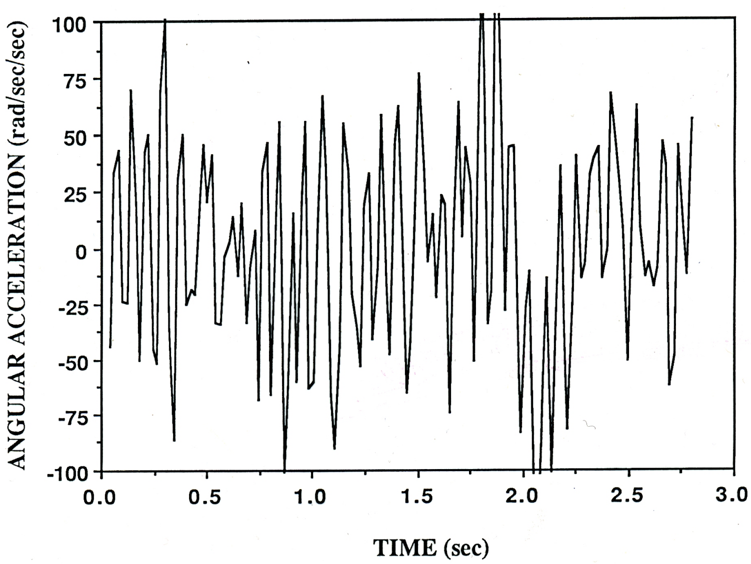
Angular Data With Noise



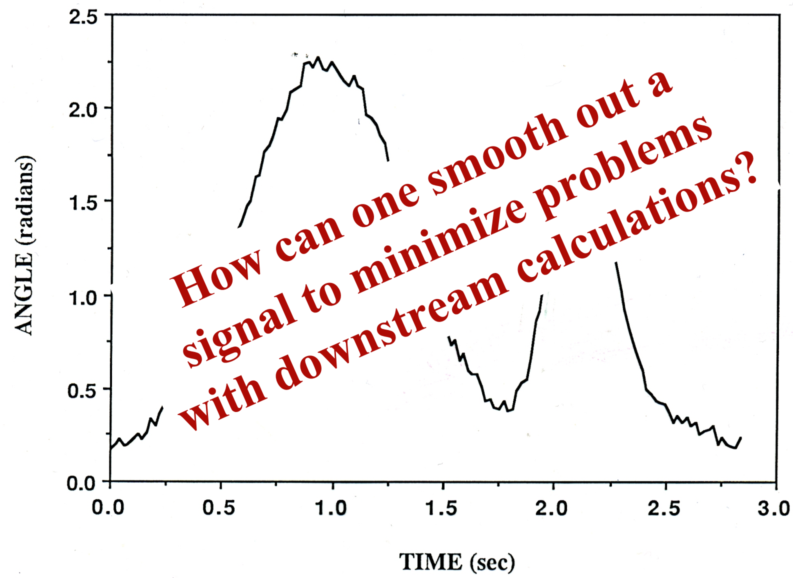
Angular Velocity



Angular Acceleration



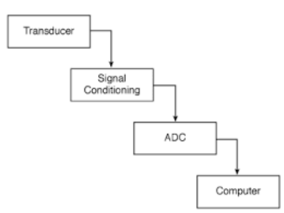
Angular Data With Noise



How can one smooth out a signal to minimize problems with downstream calculations?

Signal Amplification and Filtering

- The output of most sensors is low in magnitude, often noisy, and likely needs to be filtered and amplified.



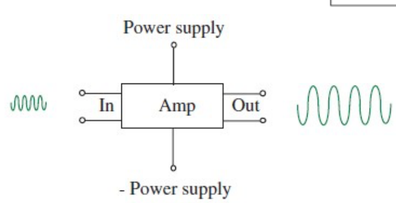


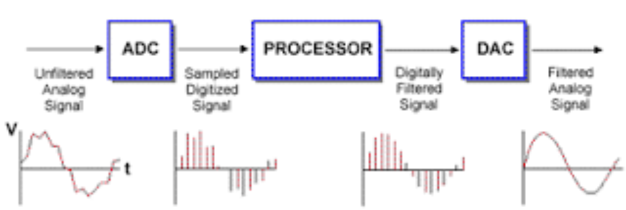
Figure 2. FPGA module installed in an industrial instrument.

Short Video on Audio Filtering

- Basic Audio Filtering

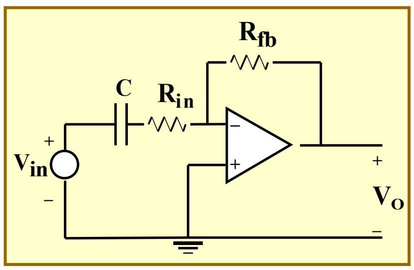
Filter Classification

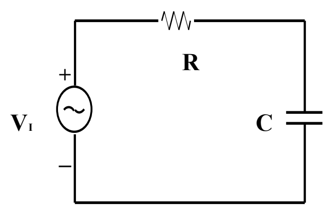
- Filters are classified as either *analog* or *digital:*
  - **Analog Filters** are either *passive* or *active* and are implemented with resistors, inductors, capacitors and operational amplifiers.
  - **Digital Filters** are implemented using a digital computer or special purpose digital hardware. A digital filter is simply the implementation of equation(s) in computer software. There are no R, L, C components as such. However, digital filters can also be built directly into special purpose computers in hardware form. But the execution is still in software.



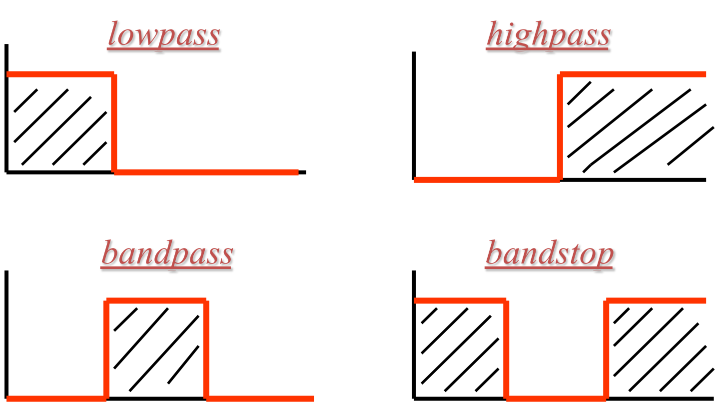
Active and Passive Filters

- An **Active Filter** is one that, along with R, L, and C components, also contains an energy source, such as an operational amplifier.
- A **Passive Filter** is one that contains only R, L, and C components. It is not necessary that all three be present. The inductor is often omitted from passive filter design because of its size and cost.

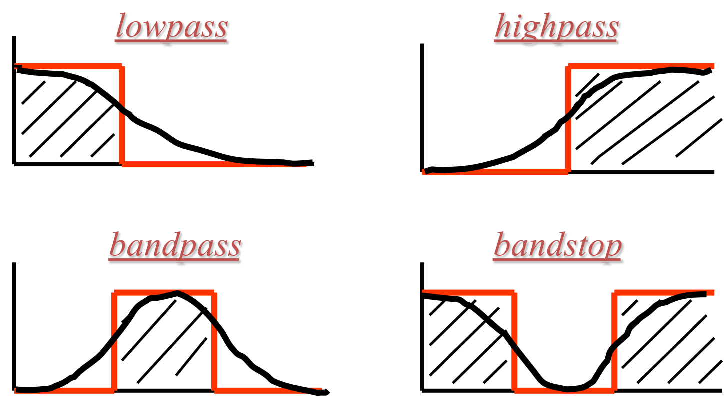




Four Types of Filters - Ideal



Four Types of Filters - Realistic



Example

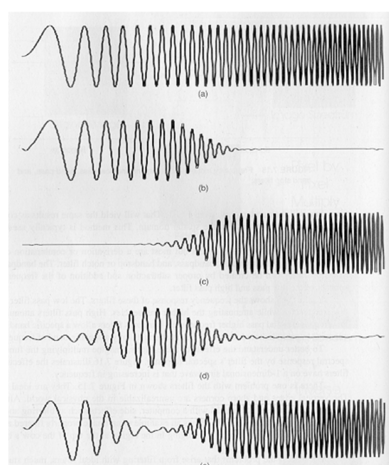
Original signal

Low-pass filtered
(Smoothing)

High-pass filtered
(Sharpening)

Band-pass filtered

Band-stop filtered



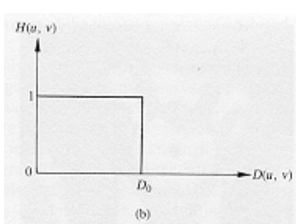
Types of Filter Implementations

- Butterworth – flat response in the passband and acceptable roll-off.
- Chebyshev – steeper roll-off but exhibits passband ripple (making it unsuitable for audio systems).
- Bessel – yields a constant propagation delay.
- Elliptical – much more complicated.
- Kalman - **Kalman filtering**, also known as **linear quadratic estimation**(**LQE**), is an algorithm that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone, by using Bayesian inference and estimating a joint probability distribution over the variables for each timeframe. The filter is named after Rudolf E. Kálmán, one of the primary developers of its theory.

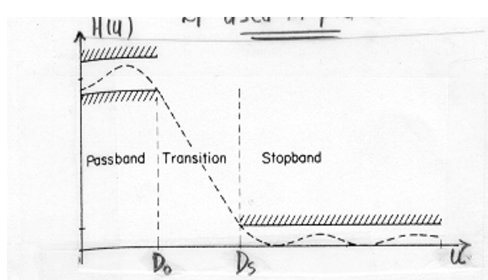
Types of Filter Implementations

- The ideal filter, sometimes called a "brickwall" filter, can be approached by making the order of the filter higher and higher.
- The order here refers to the order of the polynomial(s) that are used to define the filter.

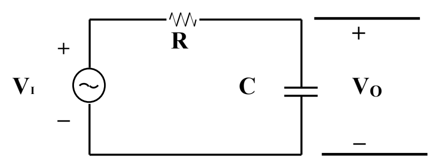
Ideal



In Practice



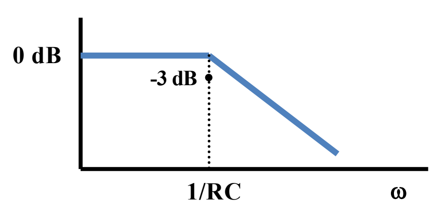
Passive Analog Low-Pass Filter



Low pass filter circuit

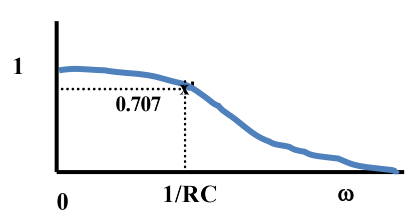
$$\frac{V_o(jw)}{V_i(jw)} = \frac{\frac{1}{jwC}}{R+\frac{1}{jwC}} = \frac{1}{1+jwRC}$$

Low-Pass Filter Frequency Response



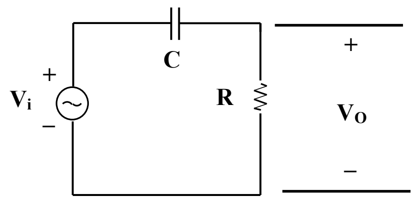
Bode

Passes low frequencies
Attenuates high frequencies



Linear Plot

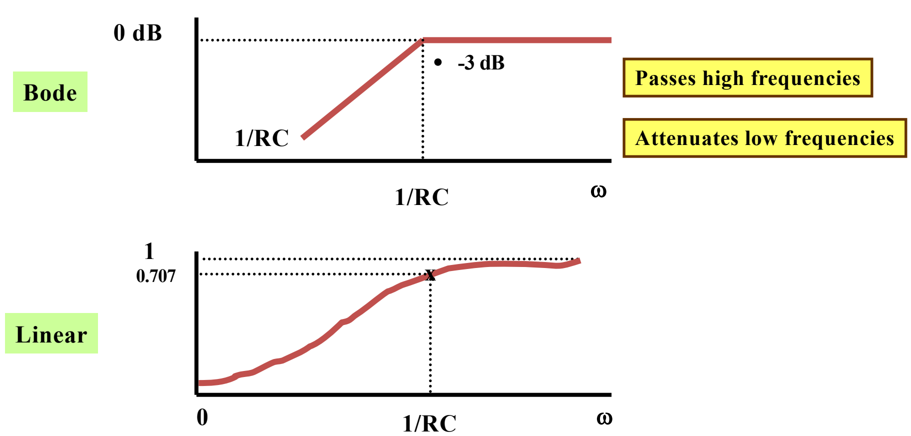
Passive High-Pass Filter



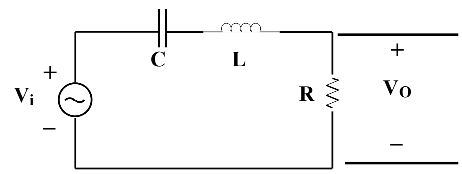
High Pass Filter

$$\frac{V_o(jw)}{V_i(jw)} = \frac{R}{R+\frac{1}{jwC}} = \frac{jwRC}{1+jwRC}$$

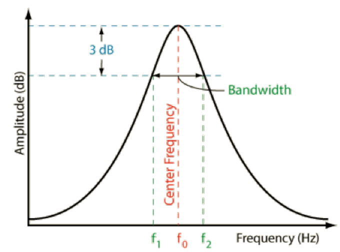
High-Pass Filter Frequency Response



Bandpass Filter

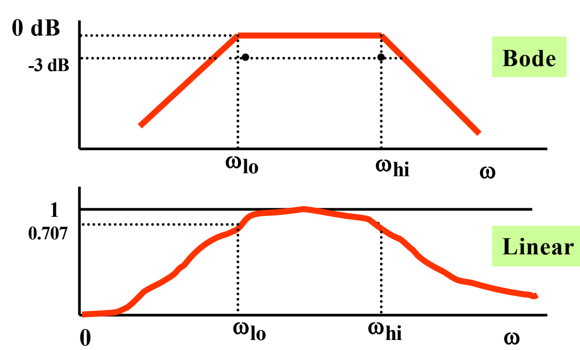


$$\frac{V_O(s)}{V_i(s)} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

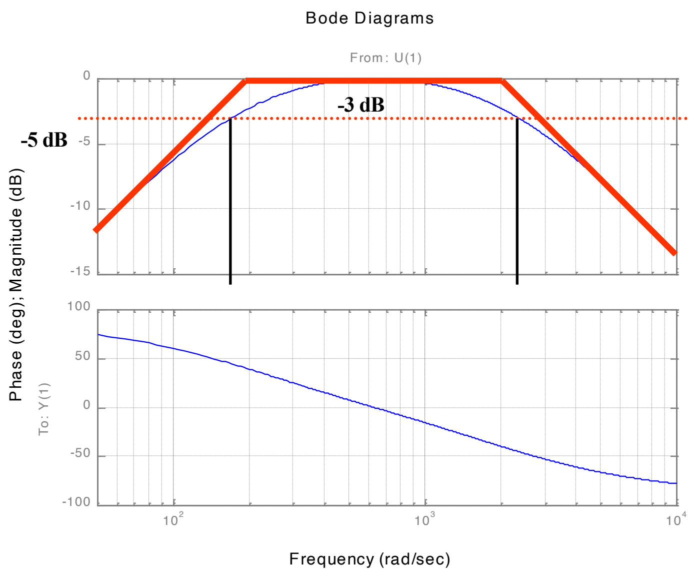


Bandpass Filter

- We can make a bandpass from the previous equation and select the poles where we like. In a typical case we have the following shapes:



Bandpass Filter

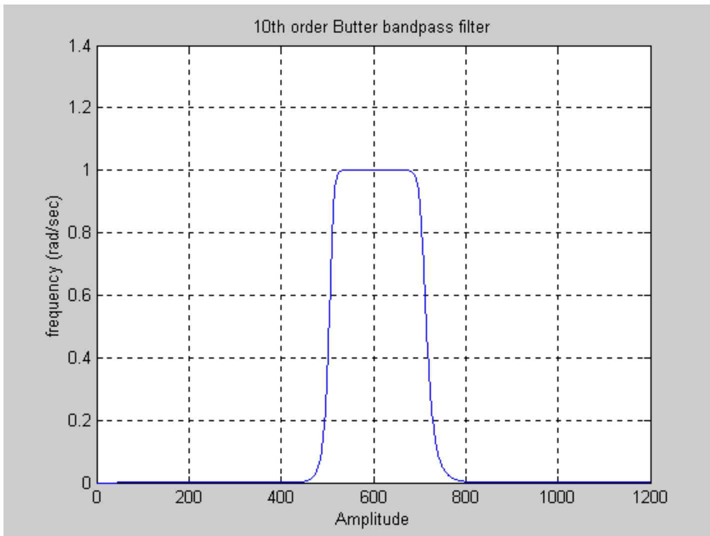


A Digital Bandpass Filter in Matlab

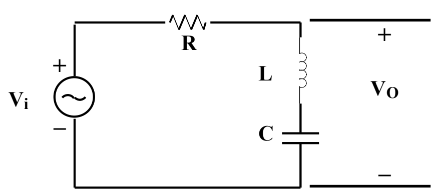
- Matlab code to simulate a 10[th] order bandpass Butterworth filter.

```
N = 10;      %10th order butterworth analog filter

[ZB, PB, KB] = buttap(N);
numzb = poly([ZB]);
denpb = poly([PB]);

wo = 600;  bw = 200;      % wo is the center freq
                          % bw is the bandwidth
[numbbs,denbbs] = lp2bs(numzb,denpb,wo,bw);

w = 1:1:1200;

Hbbs = freqs(numbbs,denbbs,w);
Hb = abs(Hbbs);

plot(w,Hb)
grid
xlabel('Amplitude')
ylabel('frequency (rad/sec)')
title('10th order Butterworth filter')
```

Bandpass Filter Frequency Plot



RLC Band-Stop Filter



- The transfer function for V_O/V_i can be expressed as follows:

$$G_v(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

RLC Band-Stop Filter

- This is of the form of a band stop filter. We see we have complex zeros on the jw axis located at:

$$\pm j\frac{1}{\sqrt{LC}}$$

- From the characteristic equation, we see we have two poles. The poles can essentially be placed anywhere in the left half of the s-plane. We see that they will be to the left of the zeros on the jw axis.

RLC Band-Stop Filter Design Example

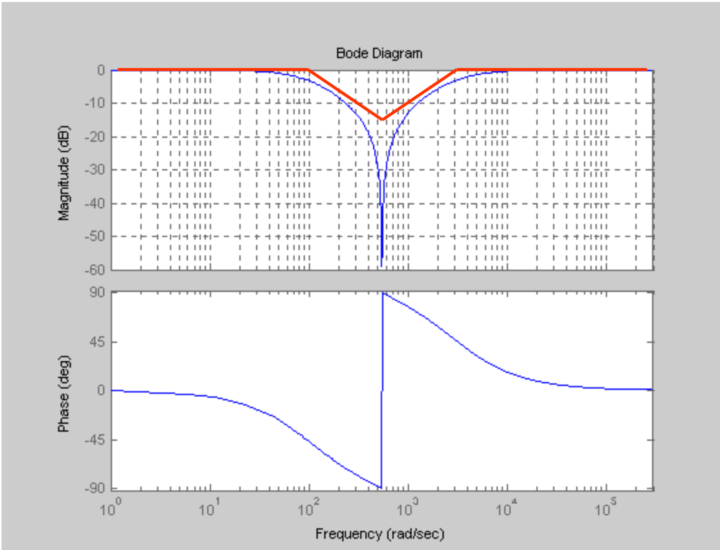
- Design a band stop filter with a center frequency of 632.5 rad/sec and having poles at –100 rad/sec and –3000 rad/sec.
- The transfer function is:

$$\frac{s^2 + 300000}{s^2 + 3100s + 300000}$$

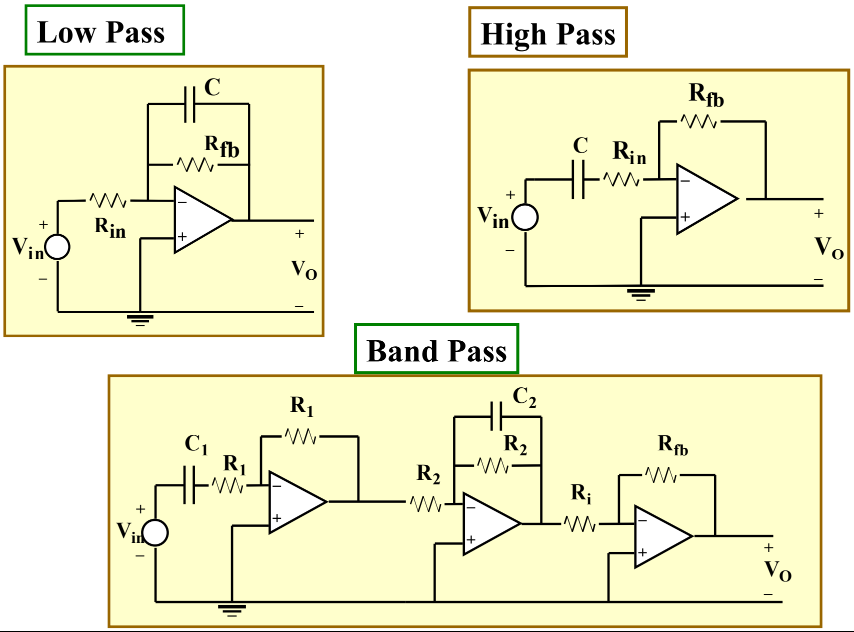
- Matlab code:

```
num = [1 0 300000];
den  = [1 3100 300000];
w = 1 : 5 : 10000;
bode(num,den,w)
```

Matlab Bode Plot

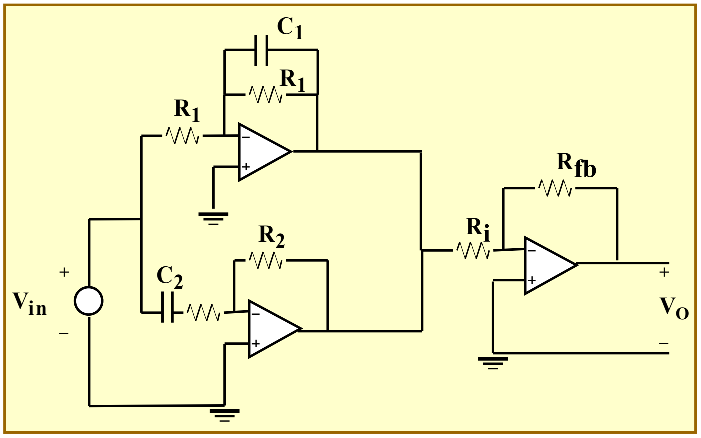


Active Filters

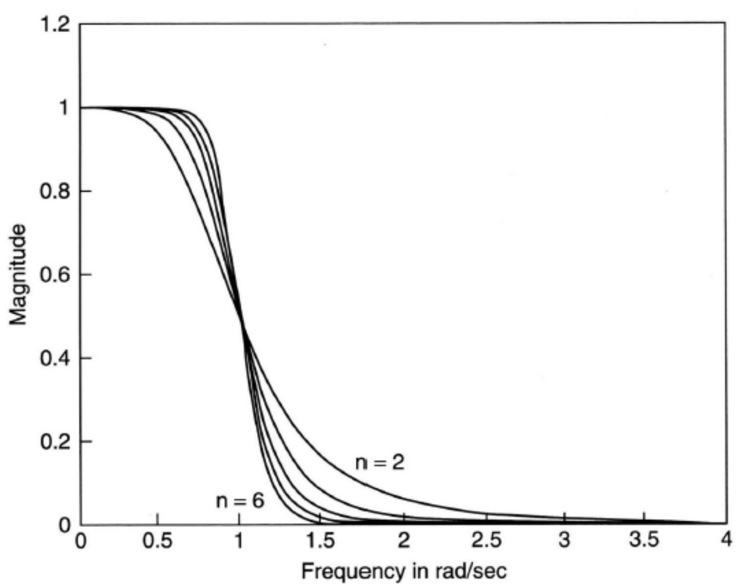


Active Filters

Band Stop



Magnitude Response of Butterworth Lowpass Filter



Butterworth Polynomials

n	Butterworth Polynomial $D(p)$ in Polynomial and Factored Form
1	$p+1$
2	$p^2+\sqrt{2}p+1$
3	$p^3+2p^2+2p+1=(p+1)(p^2+p+1)$
4	$p^4+2.61326p^3+3.41421p^2+2.61326p+1$
	$=(p^2+0.76537p+1)(p^2+1.84776p+1)$
5	$p^5+3.23607p^4+5.23607p^3+5.23607p^2+3.23607p+1$
	$=(p+1)(p^2+0.618034p+1)(p^2+1.931804p+1)$
6	$p^6+3.8637p^5+7.4641p^4+9.1416p^3+7.4641p^2+3.8637p+1$
	$=(p^2+0.5176p+1)(p^2+1.4142p+1)(p^2+1.9318p+1)$

Digital Filter Example

- Vertical Jump Video
- Excel Template for Butterworth Filter Implementation
- Excel Spread Sheet For Vertical Jump Example

Summary

- Analog or digital filters
- Active or passive filters
- Filter types
  - Low-pass, high-pass, band-pass, band-stop
- Filter methods
  - Butterworth, Chebyshev, Bessel, Kalman, etc.